Objective: The objective of this exercise is the calculation of the core and tidal radius of a globular cluster in the Milky Way. Measure the tidal radius of a globular cluster and derive an estimate of its mass.

Observations: V or R - band images of a globular cluster. More than one field may be necessary to ensure that the background is reached.
1. 10 bias frames
2. 3-5 V band sky flat fields
3. 2-3 V band CCD images of a globular cluster field

Theory topics: Globular clusters, radial profile.

Analysis: After standard bias subtraction and flat fielding of the observed fields, the student needs to find all stars visible on the frames. The student needs to find the star number density and plot it as a function of radius. Comparison against models leads to the derivation of the tidal radius and concentration parameter of the cluster. diagram (colour-colour plot) of an open cluster.
Contents: Globular Clusters

1. Globular Cluster
2. Estimating the Tidal Radius (Exercise 1)
3. The Mass of the Globular Cluster (Exercise 2)

In the following we assume that the user has loaded the IRAF and SAO image DS9 software tools. IRAF tasks are denoted with the character font ‘Century Gothic’.

Globular Clusters

1. Globular Clusters

A globular cluster is a spherical collection of stars that are very tightly bound to each other by gravity. Globular clusters orbit around the Milky Way galaxy core like satellites. The number of stars in a globular cluster varies from a few thousand up to a million stars for the more massive ones. There are ~150 known globular clusters that orbit our own Milky Way galaxy core. The stars that form globular clusters belong to a group called Population II stars and are amongst the oldest stars in the Universe with an average age of ~12 billion years. The morphology and dynamics of globular clusters can be characterized by means of their standard radii:

- **Core radius**: The distance from the center of the cluster at which the apparent surface brightness of the cluster has decreases by half.
- **Tidal radius**: The distance from the cluster core at which the gravitational influence of the galaxy is larger than that of the globular cluster core.
Fig. 1. The globular cluster M13 in the constellation Hercules. The cluster contains several hundred thousand stars, which are very densely packed together under the force of gravity.

**NOTE:** Before starting work on the exercises below, the raw CCD images have to be correctly calibrated by subtracting the BIAS and DARK currents as well as dividing by a FLAT field image. This procedure is outlined in Project 1. You should have successfully completed Project 1 before proceeding with the exercises below.

**Exercise 1: Estimate the tidal radius of a globular cluster by measuring the density of stars as a function of radius.**

The tidal radius of a globular cluster is defined as the radius at which the gravity of the galaxy has more influence on the stars of the cluster than the gravity of the cluster’s core. Thus, stars that lie outside the tidal radius tend to leave the cluster. One way of measuring this radius is by counting the density of stars as a function of distance from the cluster’s core. The distance from the cluster’s core to the point where the star density starts to approach the density of field stars is what we define as the cluster’s tidal radius.
Having obtained your final calibrated image follow these steps to measure the tidal radius of the cluster:

1. Start IRAF and DS9
2. Display the image with DISPLAY (e.g. display NGC6229.fits)
3. Find out the average stellar FWHM and the STDEV for the sky.

This can be achieved by the task imexamine in the following way:

Start the IRAF task imexamine.

To estimate the Full Width Half Maximum (FWHM) of a star place the mouse cursor in DS9 over a bright well defined star in the CCD frame. Select a star that is not in a crowded field so that it does not have nearby companions. Hit the ‘r’ key. An IRAF graphics window opens showing the radial profile of the star. At the bottom of the graphics window is a line with numbers giving basic statistics about your target. Make a note of the number that is second from left as shown in the image below. This is your estimate for the FWHM.

Fig. 2. Measuring the FWHM of a stellar CCD image.
To estimate the standard deviation of the sky, place the mouse cursor in DS9 at an area on the CCD frame that is devoid of stars and hit the m key. You should get a line in the IRAF window that looks like this:

<table>
<thead>
<tr>
<th>#</th>
<th>SECTION</th>
<th>NPIX</th>
<th>MEAN</th>
<th>MEDIAN</th>
<th>STDDEV</th>
<th>MIN</th>
<th>MAX</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SECTION</td>
<td>NPIX</td>
<td>MEAN</td>
<td>MEDIAN</td>
<td>STDDEV</td>
<td>MIN</td>
<td>MAX</td>
</tr>
<tr>
<td>25</td>
<td>[753:757,833:837]</td>
<td>25</td>
<td>731.7</td>
<td>731.6</td>
<td>6.176</td>
<td>719</td>
<td>743</td>
</tr>
</tbody>
</table>

4. Repeat step 3 a couple of times and obtain an estimate an average stellar FWHM and an average Standard Deviation for the sky. Once you have your average values hit ‘q’ in the DS9 window to exit the imexamine task.

5. Now we need to identify all the stars in our CCD frame and place a mark on them so we will be able to count them easily. To do this we will use the IRAF task DAOFIND as follows:

>`epar daofind`

You will get a list of parameters used by the DAOFIND task.

The first parameter is the file name of the image. Go ahead and change that to whatever filename you used for your calibrated image. Now using the arrow keys go down to the parameter ‘datapar’ and enter “:e” this will take you to another parameter list. Here you will need to change the following parameters:

- `fwhmsf` – enter your average FWHM
- `sigma` – enter your average Standard Deviation sky value
- `readnoi` – this is the Read Noise of the CCD camera you used to acquire your image.
- `epadu` – This is the gain of your CCD camera
- `Intime` – The exposure time that you used.

Once you are finished editing these parameters hit “:q” and this will return you to the parameter list of the DAOFIND task. Hit “:q” again and you go back to the IRAF prompt. Now enter:

>`daofind`

Hit enter to accept the default parameters that you set. When the task asks you for the Detection Threshold in Sigma enter 10. This basically defines the sensitivity of the star detection algorithm. If you set it too high you will only detect the brightest stars, if you set it too low you will have fake detections due to random sky fluctuations, bad pixels etc.
Having ran the task DAOFIND you will see that a file called “filename.fits.coo.1” was created. This file holds the image coordinates of all the stars in the CCD frame.

6. Place a mark on the stars that DAOFIND detected so we can easily count them.

This can be done with the IRAF task TVMARK. Enter the following command at the IRAF prompt:

```
> tvmark frame=1 coords=filename.fits.coo.1 pointsize=15
```

You should now have rectangular points over all or most of the stars in your DS9 window.

7. To measure the star density of the cluster as a function of distance from the stars core we need to divide the image in circular rings centered at the cluster’s core. Start by creating a circular region in DS9.

Hold the left button anywhere on the DS9 image and drag the mouse. This creates a circular region whose size increases as you drag your mouse. Having created a circular region double click inside the region. This opens up a window with the region’s properties as is shown in Figure 3.

![Fig. 3 The Circular Region dialog box of DS9.](image)

All of the rings that we will construct need to be concentric and increase in radius with a constant value. In this example we use 15 rings with each ring having a radius of 15 pixels longer than the previous one. Once you finished creating your rings you should have an image similar to the left image of Fig. 4.
8. Now, for each ring count the number of stars in the ring. For stars that lie exactly on a ring’s edge is up to you to decide at which ring the star belongs to. Just remember to only count these stars once. Fill in Table 1 with your results. The error in your number of stars in each ring is defined as:

\[ \Delta N = \sqrt{N} \]

Where \( N \) is the number of stars in each ring.

Using the pixel scale of your CCD camera (e.g. 2 arcseconds/pixel for Skinakas) convert your ring radii from pixels to arcseconds.

You will also need to calculate the number of stars per unit area. The area of each ring is given by:

\[ A = \pi(r_{\text{out}}^2 - r_{\text{in}}^2) \]

9. Make a plot of Number of Stars per unit area Vs Radius in arcseconds. Don’t forget to include error bars in your plot.

10. In your plot you will find that the number of stars per unit area decreases with increasing radius and in the outer radii it levels off and reaches a constant value that corresponds to the number of background stars. Estimate this constant value from your plot and draw a straight line indicating this background. The tidal radius can
then indicated by the intersection of your star density distribution with this line indicating the background. If you find that the distribution of star density has not leveled off consider adding more rings to the outer edges of the cluster.

Estimate the uncertainty of your value for the tidal radius. Comment on the factors that contribute to the uncertainty of your value.

Table 1. Fill the table with the number of stars found within each ring.

<table>
<thead>
<tr>
<th>Ring #</th>
<th>Radius in Pixels</th>
<th>Radius in arcseconds</th>
<th>Number of Stars in ring</th>
<th>Number of Stars per unit area</th>
<th>Error in the Number of Stars per unit Area</th>
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<tbody>
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Exercise 2: Calculate the mass of the globular cluster.

Having measured the tidal radius of the globular cluster you are now in a position to calculate its total mass.

1. Using the following formula calculate the mass of the globular cluster:

\[ r_i = R \left( \frac{M_{\text{cluster}}}{2M_{\text{galaxy}}} \right)^{1/3} \]
In the above equation $r_t$ is the tidal radius, $R$ is the distance of the cluster from the galactic center and $M_{\text{cluster}}$ and $M_{\text{galaxy}}$ are the masses of the globular cluster and the Milky Way galaxy in units of solar masses $M$.

You can assume a mass for the Milky Way galaxy of: $M_{\text{galaxy}} = 6.0 \times 10^{11} M$.

A reference catalogue with distances to all the known globular clusters as well as their measured dynamical parameters can be found at:

http://www.physics.mcmaster.ca/~harris/mwgc.dat

Also, calculate the uncertainty of your mass estimate by propagating your error estimate for the tidal radius.

2. Using the above catalogue compare your estimate of the tidal radius with the one quoted in the catalogue. Comment on how well your estimate agrees with the value from the catalogue.