

# PROJECT

## 4

### THE HERTZSPRUNG – RUSSELL DIAGRAM OPEN CLUSTERS

**Objective:** The aim is to measure accurately the B and V magnitudes of several stars in the cluster, and plot them on a Colour Magnitude Diagram. The students will be asked to identify the Main Sequence of the stars in the cluster and measure their temperature and mass.

**Observations:** This exercise requires the acquisition of two images, in B and V filters, of an open cluster.

**Theory topics:** Open clusters, HR diagram, colour-colour diagram, stellar evolution, spectral type and luminosity class of stars.

**Analysis:** Create an HR diagram (colour-colour plot) of an open cluster. Determine the position of different type of stars in the diagram and their physical properties.

## Contents: The Hertzsprung-Russell Diagram

### Open Clusters

1. Definition
2. Interpretation
  - 2.1. Main Sequence
  - 2.2. Giants & Supergiants
  - 2.3. White dwarfs
3. Construction. Open Clusters

## The Hertzsprung-Russell Diagram Open Clusters

### 1. Definition

The Hertzsprung-Russell Diagram or HR diagram gets its name from the two astronomers that first produced it. In the early 1900's, Ejnar Hertzsprung and Henry Norris Russell independently made the discovery that the luminosity of a star is related to its surface temperature. Luminosity is a measure of how much energy a star gives off. So, essentially, the HR diagram graphed how much energy a star gives off as a function of the star's temperature.

From the teaching project 3 you know that the colour of a star is related to the temperature of that star. You may also know that the spectral classification also gives an indication of the temperature of the star. The current system of naming spectral class was adopted in 1910 and consists of a letter and a number from 0 to 9, for example the spectral class of the Sun is G2. The letters used are in decreasing order of temperature

O B A F G K M

Class	Temperature	Star colour
O	<u>30,000 - 60,000 K</u>	Bluish ("blue")
B	<u>10,000 - 30,000 K</u>	Bluish-white ("blue-white")

A	<u>7,500 - 10,000 K</u>	White with bluish tinge ("white")
F	<u>6,000 - 7,500 K</u>	White ("yellow-white")
G	<u>5,000 - 6,000 K</u>	Light yellow ("yellow")
K	<u>3,500 - 5,000 K</u>	Light orange ("orange")
M	<u>2,000 - 3,500 K</u>	Reddish orange ("red")

Thus, the horizontal axis in a HR diagram can be effective temperature, colour indices or spectral class, while the vertical axis can be luminosity with respect to that of the Sun or the absolute magnitude  $M_V$ .

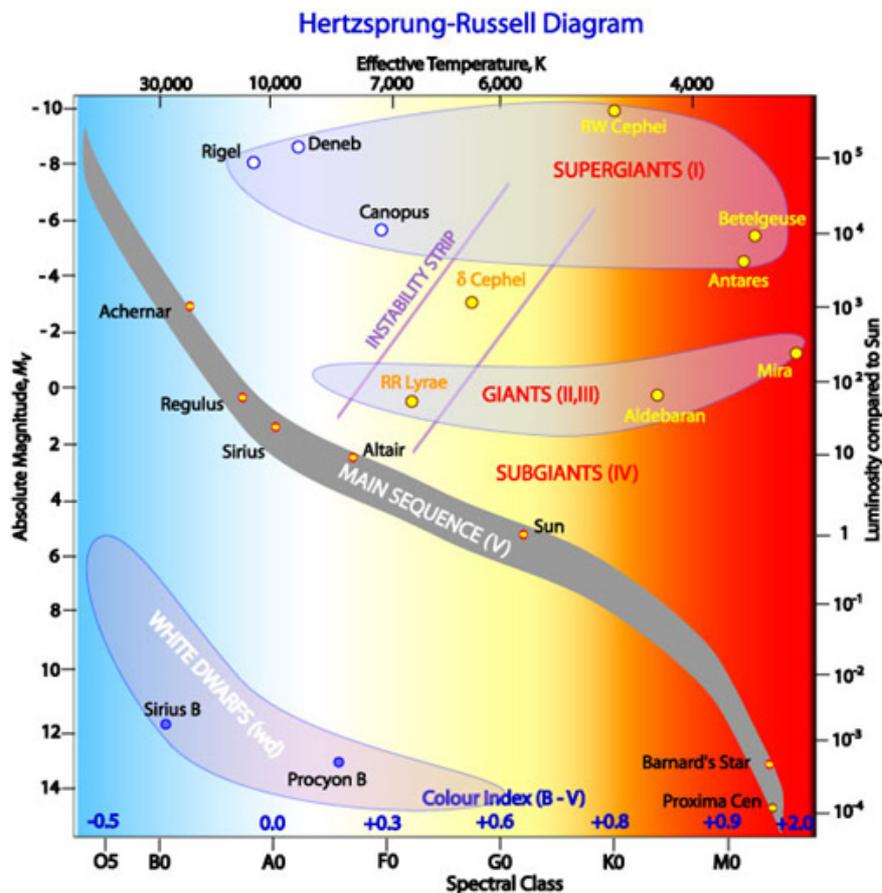


Fig. 1. The Hertzsprung-Russell diagram. From the Australia Telescope outreach and education site.

When luminosity is plotted as a function of the temperature for a large number of stars, stars do not fall randomly on the graph; rather they are confined to specific regions. This tells you that there is some physical relationship between the luminosity and temperature of a star. From the figure, one sees that most stars fall along a diagonal strip from high temperature, high luminosity stars to low temperature, low luminosity stars. These are the **main sequence** stars. Our Sun is one of them. There are a few stars that are not in this diagonal strip. There are some low temperature, high luminosity stars - these are called **giants** and **supergiants**. The reason they are so luminous while being relatively cool is because they are so big (50 times more massive than our Sun). Another group of stars are in the high temperature, low luminosity corner of the diagram. Since these stars are hot, but not very luminous, they must be very small, so they're called **white dwarfs**.

**Exercise 1: Find out the position in the HR diagram and the nature of the following stars: Antares, Proxima Centauri Rigel, Betelgeuse, Aldebaran, the Sun.**

The Sun: G2v (G2 Main-Sequence star)

Betelgeuse: M2Ib (M2 Supergiant star)

Rigel: B8Ia (B8 Bright Supergiant star)

Sirius: A1v (A1 Main-Sequence star)

Aldebaran: K5III (K5 Giant star)

## 2. Interpretation

### 2.1. Main Sequence

Most nearby stars (85%), including the Sun, lie along a diagonal band in the H-R diagram called the **Main Sequence**.

Ranges of stellar properties:

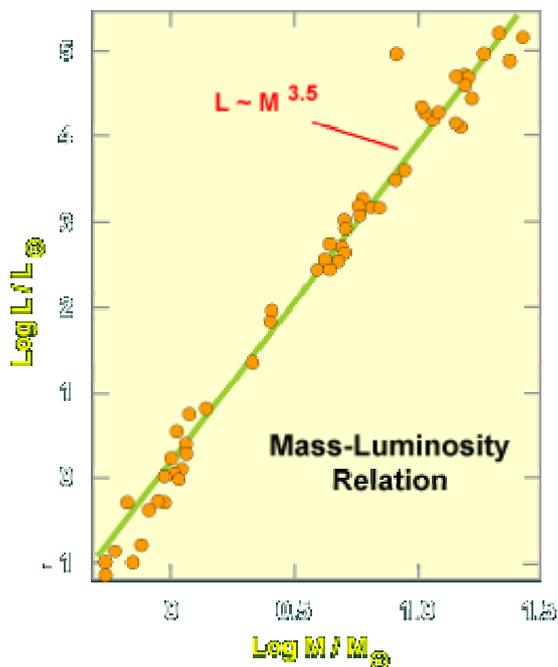
- $L=10^{-2}$  to  $10^6 L_{\text{sun}}$
- $T=3000$  to  $>50,0000$  K
- $R=0.1$  to  $10 R_{\text{sun}}$

- **The Mass – Luminosity relation**

For main sequence stars, the luminosity increases with the mass with the approximate power law:

$$\frac{L}{L_{\odot}} = \left[ \frac{M}{M_{\odot}} \right]^{3.5}$$

The expression uses luminosities and masses compared to those of the Sun.



As shown in the figure, the brightness of Main-Sequence stars varies proportional to some power of their masses. For most of the range of stellar masses, the proportionality is as the 3.5 power of the mass.

Fig. 2. Mass-Luminosity relation for Main Sequence stars.

**Exercise 2: Sirius is about twice as massive as the Sun. How much brighter is it with respect to the Sun? In the HR diagram, where will you find the least massive stars? and the most massive stars?**

If I double the mass of a main sequence star, the luminosity increases by a factor  $2^{3.5} \sim 11.3$ . So, Sirius is more than 10 times brighter than our Sun.

The luminosity of a star is thus a very strong function of its mass. Using the Mass-Luminosity relation, we infer that the least massive stars are at the lower right hand part of the Main Sequence and that the most massive stars are at the upper left hand part of the Main Sequence. The Sun, a G2V star sits around the middle of the Main Sequence.

The fact that the luminosity is such a strong function of the mass of the star, has great implications for how long stars live on the Main Sequence. Massive stars have very short lifetimes compared to the Sun (which has a lifetime on the order of 10 billion years or so).

A star's lifetime on the main sequence is how long it takes to use up the hydrogen in its core. The luminosity (L) of a star is a measure of how rapidly it is using up its hydrogen. The mass (M) of a star is a measure of how much fuel it has. The time it takes to use up the fuel is proportional to its amount of fuel (M) divided by the rate of fuel consumption (L),

$$t \approx \frac{M}{L}$$

where, t = lifetime (in units of the Sun's lifetime)

M = mass (in units of the Sun's mass)

L = luminosity (in units of the Sun's luminosity)

Since  $L \sim M^{3.5} \rightarrow t \sim M/L \sim M/M^{3.5} \sim 1/M^{2.5}$

$$t \approx \frac{1}{M^{2.5}}$$

**Exercise 3: Estimate the lifetime of a star with  $M=0.2 M_{\text{sun}}$  and  $M=50M_{\text{sun}}$ . Assume that the Sun has a lifetime of the order of 10 billion years.**

$$t = 1 / (0.2)^{2.5} t_{\text{sun}} = 56 t_{\text{sun}} = 560 \text{ billion years}$$

$$t = 1/50^{2.5} t_{\text{sun}} = 0.0006 \text{ billion years}$$

Similar calculations can be done for stars of other masses. Very hot and luminous stars (of spectral type O and 'B') have short lifetimes; if you see an O or B main sequence star you know it must be much younger than the Sun. However, cool and dim stars (of spectral type K or M) have long lifetimes; if you see a K or M main sequence star it might be older than the Sun.

- **The Luminosity – Radius – Temperature relation**

"The Luminosity of a star is proportional to its Effective Temperature to the 4<sup>th</sup> power and its Radius squared."

$$L = 4\pi R_*^2 \sigma T_*^4$$

$\sigma$  is the Boltzmann's constant.

**Exercise 4: Two stars have the same size ( $R_A=R_B$ ), but star B is 2 times hotter than star A ( $T_B=2T_A$ ). Which star is brighter? How much brighter?**

For star A:  $L_A = 4\pi R_A^2 \sigma T_A^4$

For star B:  $L_B = 4\pi R_A^2 \sigma (2T_A)^4 = 16L_A$

Therefore: Star B is  $2^4$  or 16x brighter than Star A.

*\*If two stars are the same size, the hotter star is brighter\**

**Exercise 5: Two stars have the same effective temperature, ( $T_A=T_B$ ), but star B is 2 times bigger than star A ( $R_B=2R_A$ ). Which star is brighter? How much brighter?**

For star A:  $L_A = 4\pi R_A^2 \sigma T_A^4$

For star B:  $L_B = 4\pi (2R_A)^2 \sigma T_A^4 = 4L_A$

Therefore, Star B is  $2^2$  or 4x brighter than Star A.

*\*If two stars have the same effective temperature, the larger star is brighter\**

## 2. 2. Giants & Supergiants

There are also two bands of stars in the H-R diagram that are brighter than Main Sequence stars with the same effective temperatures.

The Luminosity-Radius-Temperature relation tells us that the stars in these bands must therefore be larger in radius than Main Sequence stars.

There are two groups of giant stars:

**Giants:** Large but cool stars with a wide range of luminosities:

- $R = 10$  to  $100 R_{\text{sun}}$
- $L = 10^3$  to  $10^5 L_{\text{sun}}$

**Supergiants:** The very largest stars, arranged along the top of the H-R diagram with a wide range of effective temperatures but relatively narrow range of luminosities:

- $R > 10^3 R_{\text{sun}}$
- $L = 10^5$  to  $10^6 L_{\text{sun}}$

### 2. 3. White Dwarfs

There are also a few very hot but also very faint stars that occupy the lower left-hand corner of the H-R Diagram. These are stars that are much fainter than Main Sequence stars of the same temperature.

The Luminosity-Radius-Temperature relation tells us that these stars must therefore be smaller in radius than Main Sequence stars.

How small? Using the Luminosity-Radius-Temperature relation, we can make a prediction:

- $R \sim 0.01 R_{\text{sun}}$

This is about the size of the Earth!

These stars are called White Dwarfs. "White" because they tend to be very hot ("white hot") and "Dwarfs" because they are so tiny.

### 3. Construction. Open Clusters

#### ***Exercise 6: Create a colour-magnitude diagram of an open cluster.***

The objective of this project is to learn how to produce colour-magnitude diagrams of star clusters using CCD images obtained with the CCD system at our observatory. The technique involves getting images of the cluster through two filters, B and V in our case, reducing the data and plotting V versus (B-V).

In some circumstances, such as when plotting stars in a specific open or globular cluster, apparent magnitude  $m$ , or  $V$ , rather than absolute magnitude may be used in the Y axis of the HR diagram. This is valid as all the stars in the cluster are effectively at the same distance away from us hence any differences in apparent magnitude are due to actual difference in luminosity or  $M$ . Diagrams where  $V$  is plotted against colour index, B-V, are also known as *colour-magnitude diagrams*.

In order to avoid the complication of using absolute photometry and be less weather-dependent we will select clusters with secondary standards, that is, constant stars with reliable values of the B and V magnitudes. As these "secondary standards" will be in the same image, all stars are observed at the same airmass, hence, the derivation of the atmospheric extinction can be omitted. The transformation of the instrumental magnitudes into absolute magnitudes is simplified, as the product of the extinction coefficient times the airmass is a constant, that will be absorbed by the coefficients in the

transformation equations. The final magnitudes are used to create an H-R diagram by representing V as a function of the colour index B-V.

1. Calculate a mean bias frame and subtract the bias from the B and V images.
2. Flat-fielding: Take all the flat-field images taken with the same filter, add them together and divide the sum by the mean value of the sum. This will give an averaged flat-field whose mean is unity. Divide the images by the scaled flat-field image.
3. Obtain the instrumental magnitudes of at least 70 stars. Note that long exposure times will probably overexpose the brightest stars, so you will have to take another pair of short exposures that does not overexpose them.
4. Transform to the standard system. This will involve finding published values of V and B for some of the stars in your cluster and also a finding chart so that the stars with published values can be identified on your images. There is a great resource to help with this on the web:

<http://www.univie.ac.at/webda/Welcome.html>

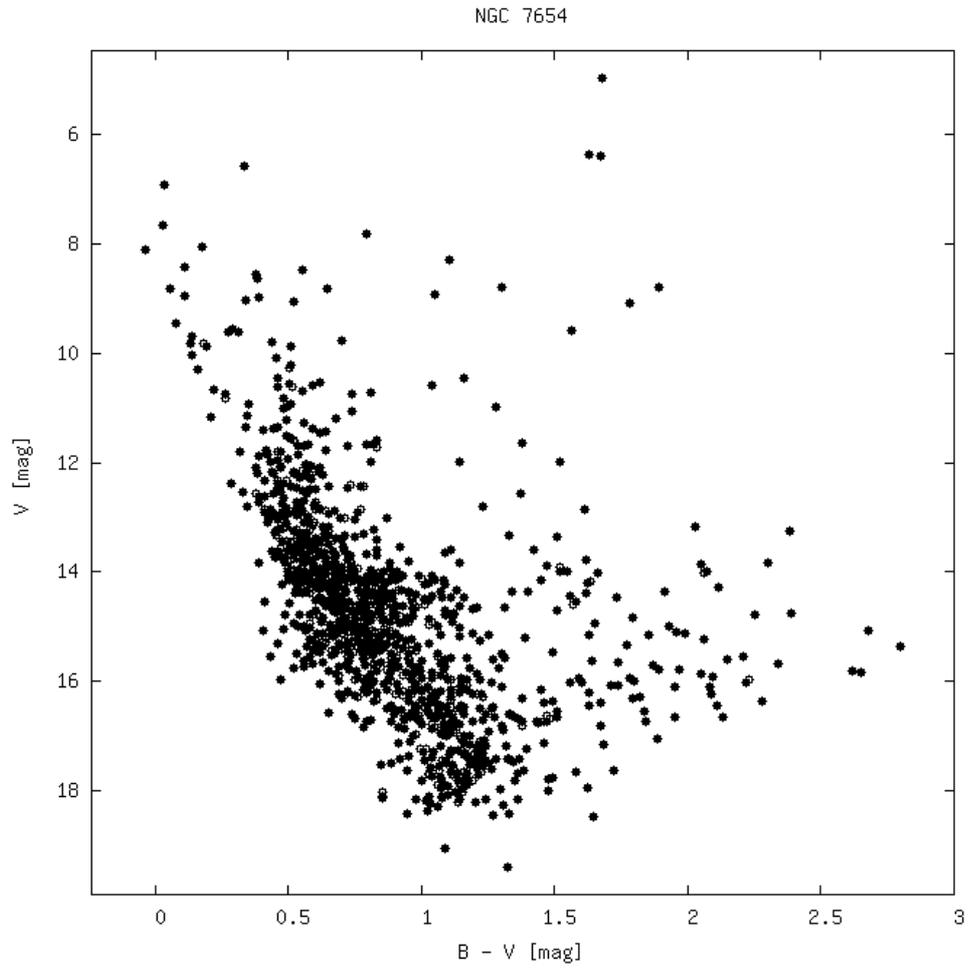
[http://www.univie.ac.at/webda/cgi-bin/ocl\\_page.cgi?dirname=ngc7654](http://www.univie.ac.at/webda/cgi-bin/ocl_page.cgi?dirname=ngc7654)

This site has collected almost all the available data on open clusters and you can call up a plot showing the cluster you have measured. You can scale this plot to show the stars you see in your image, and then, by just clicking on a particular star, get its published values of V and (B-V) – provided data on that star exists. All you need is two stars, but you will want to make sure they have a large difference in colour. If one has standard stars in the science images you can skip the atmospheric corrections, and go right to the transformation equations. This means determining a transformation constant  $t_0$  such that:

For the standard stars:  $t_0 = m_{\text{std}} - m_{\text{inst}}$

For the program stars:  $m = t_0 + m_{\text{inst}}$

5. Plot up the H-R diagram using your data transformed to the standard V, (B-V) system.
6. Compare your results to published H-R diagrams. Your diagram of M52 should be similar to the one shown below.



**Fig. 3.** H-R diagram of M52 taken from the WEBDA database.